Mean Life of the Neutral π Meson*

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We report an experiment to measure the π^0 mean life, using a new method that utilizes the relativistic flight-path dilation produced by a high pion velocity. A π^- beam of 3.5 BeV/c made interactions in Ilford K.5 emulsion. Some 103 secondary neutral pions were observed to decay by the Dalitz mode ($\pi^0 \rightarrow e^+ + e^- + \gamma$) in a study of 3600 such stars. The π^0 path lengths could be accurately measured and were automatically calculated by specially developed equipment. We assumed the π^0 momentum spectrum to be the same as that of the secondary charged pions found by multiple-scattering measurements. The mean transverse momentum of secondary charged pions at all angles is 274 ± 10 MeV/c. The opening-angle distribution of the electron pairs was measured and compared with extensive calculations of the expected laboratory-system distribution in order to evaluate the number of unobserved events. We used a statistical approach to evaluate the mean life (τ_{π^0}) of the π^0. To evaluate the complex effects of electron scattering, the finite grain size, and the random grain spacing, pairs of electron tracks were generated artificially by a Monte Carlo program and the results compared with the observations. Our estimate of the neutral-pion mean life is $(1.7\pm0.5)\times10^{-16}$ sec.

I. INTRODUCTION

WE recently described a new method for measuring the mean life of the neutral pion and presented preliminary measurements.[1] This is a report of the completed work. A large amount of new data has been added, and careful studies of the systematic errors have been made. Consequently, we now are able to quote a result with relatively satisfactory confidence limits.

In common with the $K_{\pi2}$ method,[2–7] we determine the points of decay of the mesons by observing the origins of Dalitz pairs[8] in fine-grain emulsion.

Our experiment otherwise is quite different. It includes a number of innovations that were introduced in order to increase the pion flight paths, and to improve the objectivity and accuracy of the measurements. Two difficulties of the $K_{\pi2}$ method are (a) uncertainty in the actual point of origin of the π^0, and (b) the extremely small flight path of the π^0. Each was studied, and the new experiment was designed to overcome them. We produce the π^0 mesons in Ilford K.5 emulsion by collisions of 3.5-BeV/c π^- mesons with emulsion nuclei. Typically several tracks of high-energy secondary particles are then produced in the interaction, and their common point of origin defines very well the π^0 origin. In such a high-energy collision, too, the π^0 is produced with an average momentum of 812 MeV/c—about four times that acquired in $K_{\pi2}$ decay. The average flight path is increased correspondingly. It might be thought that the opening angles of the Dalitz pairs would be reduced in inverse proportion, but at this energy the average opening angle actually remains almost unaffected (see the Appendix).

An important feature of our method has been the development of automatic equipment for measurement and data reduction. With its help we obtain results of increased precision and avoid subjectivity in the analysis.

In gaining these advantages we were forced to sacrifice an excellent feature of the $K_{\pi2}$ method. In the decay of the K meson by the two-pion mode, the π^0 momentum vector is accurately known. In our method we must assume that the distribution function of the π^0 momenta is the same as that of the charged pions. We then measure the charged-pion spectrum by multiple-scattering measurements.

A third method[9–13] for determining the π^0 mean life, first suggested by Primakoff, is to use the inverse process of its 2γ decay, which is the photoproduction of π^0 by the Coulomb field of a nucleus. Recently, Sona suggested a modification of Primakoff's method,[14] the difference being that the Coulomb field of Sona's method is provided by an electron.

In regard to the Primakoff method, it has been pointed out that in addition to the lifetime effect there are other possible influences at small angles.[15] These

* This work was done under the auspices of the U. S. Atomic Energy Commission.
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[15] G. Davidson, Ph.D. thesis, Massachusetts Institute of Technology Report TR-72, 1959 (unpublished).

additional unknowns occur even at small values of momentum transfer to the nucleus where the lifetime effect is greatest. One process contributing to these deviations is the inelastic production at forward angles. A contributing effect also may very well be the production of multipion states. Another difficulty with the Primakoff method has been the low intensity of the high-energy incident photons. Because that method depends very heavily on theory, we now believe a direct measurement to be more reliable, at least until all features of the process are better understood.

II. EXPERIMENTAL PROCEDURE

Plates of 600-μ Ilford K.5 nuclear research emulsion were exposed to the 3.5-BeV/c negative pions of the Bevatron. The total intensity was approximately 3×10^5 cm^{-2}. The fractional momentum spread was ±0.15 (full width at half-maximum).

Procedures for scanning, measurements, and data reduction have been given in our preliminary report.[1] The center of the primary π^- interaction is found with a typical error of 0.03 μ. The point of decay of the π^0 (by the Dalitz mode) was taken to be the intersection of the lines of flight of the electrons. Reference 1 further discusses the detection of neutral pions decaying by the Dalitz mode. As described in the same reference, the momentum spectrum of the secondary charged pions was found by measuring the multiple scattering of the tracks.

III. EXPERIMENTAL RESULTS

A. Data

All our data are included in the present report. Some 3600 negative pion stars were measured. Associated with these we found 103 (Dalitz) electron pairs. Only the projection of the π^0 path on the plane of the emulsion could be determined with accuracy. By the process described in reference 1, only those π^0 mesons emitted in a forward cone of approximately 60 deg half-angle were selected for the mean-life determination.

Figure 1(a) shows the distribution of the projected angles of the neutral pions. The projected angle distribution of the *charged* secondary pions is shown in Fig. 1(b). The similarity between the projected-angle distributions of the charged and neutral pions is satisfactory.

Table I lists all the Dalitz pairs in order of increasing opening angle. At high pion energies, some opening angles are increased and others decreased from their c.m. values. Projected path lengths or "gaps" of all measurable neutral pions and the projected angles of emission relative to the primary negative pion are also given. For a gap observation to be considered a measurement, the gap length had to exceed the error in its measurement by a factor of 3.5.

The errors quoted in Table I are approximations

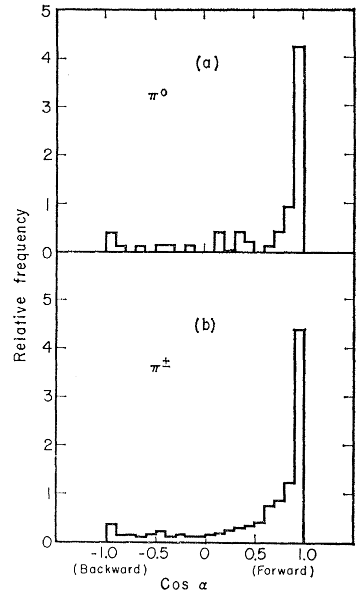

FIG. 1. Projected angular distributions of (a) secondary neutral pions, and (b) charged pions. In (a), only the *measured* events are included (see references to Table I).

based on least-squares fitting of intersecting straight lines to the coordinates of grain centers. A more correct treatment of the over-all statistical error is given in Sec. B.

Among the events that were missed or unmeasured were the following types:

(a) Events with only two secondary minimum-ionizing tracks. Another secondary is needed to define the star center; therefore, no measurement could be made. We observed ten stars of this type.

(b) Events with a very close (e^+,e^-) pair. When the product of the angle between the tracks and the gap becomes less than about 3.44 μ-deg, the error is too large for accurate measurement of the gap. We observed 18 events of this type.

(c) Pairs of large opening angle but with very small gaps, which lie within the "circle of confusion" around the star center.

We measured 75 π^0 events at all angles relative to the incident π^-. An additional 28 events were observed which were of types (a) and (b) above. Within the 60-deg cone, the corresponding numbers were 56 (measured) and 19 (additional observed). The number of events of type (c) (which were missed) will be estimated later from the theoretical distribution in the laboratory system of the opening angles of the Dalitz pairs.

Figure 2 shows the opening angles (in the laboratory system) of all observed Dalitz pairs. The smooth curve is the calculated distribution from the assumed momentum spectrum of the π^0 and the opening angle distribution in the rest system of the π^0 as calculated

TABLE I. List of observed Dalitz pairs (in order of increasing opening angle).[a]

Event No.	Projected π^0 path length s (μ)	Projected angle of π^0 path relative to incident π^- (deg)	Opening angle of pair in laboratory system (deg) α
V4 -213	1.86±1.69[b]	[d]	1.3
V8 -101	0.98±1.42[b]	[d]	1.5
V6 -212	1.05±0.98[b]	[d]	1.6
V10-3	0.80±1.33[b]	[d]	1.9
V6 -38	1.02±0.96[b]	[d]	2.0
V4 -104	0.32±0.56[b]	[d]	2.3
V2 -103	[c]	[d]	2.3
V2 -144	0.49±0.62[b]	[d]	2.4
V11-186	[c]	[d]	2.4
V4 -82	0.52±0.61[b]	[d]	2.8
V3 -91	0.63±0.70[b]	[d]	2.8
V9 -109	0.58±0.72[b]	[d]	2.8
V6 -352	2.31±0.34	20.9	2.9
V6 -105	0.39±0.60[b]	[d]	3.1
V6 -41	0.57±0.53[b]	[d]	3.2
V4 -364	0.35±0.42[b]	[d]	3.5
V7 -237	0.54±0.55[b]	[d]	3.5
V7 -184	[c]	[d]	3.5
V3 -24	0.40±0.67[b]	[d]	4.1
V2 -79	[c]	[d]	4.1
V6 -253	0.49±0.32[b]	[d]	4.2
V3 -3	0.80±0.31	18.5	4.3
V4 -322	1.37±0.27	25.1	4.4
V3 -83	1.25±0.19	35.1	4.7
V2 -124	1.44±0.25	5.2	4.7
V9 -132	[c]	[d]	5.5
V11-23	0.50±0.42[b]	[d]	5.6
V10-41	[c]	[d]	6.0
V3 -41	1.05±0.17	14.1	6.0
V6 -572	1.99±0.03	114.9	6.0
V3 -102	0.38±0.36[b]	[d]	6.4
V2 -84	1.06±0.16	159.6	6.9
V6 -86	1.03±0.14	142.3	7.3
V11-203	0.93±0.16	3.1	7.7
V8 -102	1.87±0.18	21.7	7.8
V3 -202	[c]	[d]	8.0
V3 -9	[c]	[d]	8.2
V6 -298	0.59±0.11	16.5	8.3
V6 -274	2.16±0.10	29.8	8.4
V7 -148	0.77±0.07	8.6	8.8
V6 -184	0.68±0.14	18.5	8.9
V4 -312	[c]	[d]	9.2
V10-84	[c]	[d]	9.4
V10-14	0.51±0.12	4.1	9.4
V10-177	1.01±0.05	25.5	9.5
V6 -228	0.98±0.08	9.3	9.6
V2 -114	1.30±0.15	84.0	10.2
V6 -248	0.64±0.11	26.2	10.3
V10-39	0.37±0.10	16.3	10.5
V7 -18	0.55±0.04	23.7	10.9
V10-104	0.34±0.09	50.6	11.2
V12-111	0.40±0.14	22.6	11.3
V11-143	0.81±0.03	2.8	11.8
V2 -244	0.37±0.05	61.9	12.9
V3 -72	0.51±0.09	38.1	13.1
V6 -94	0.49±0.11	19.2	13.2
V4 -149	0.72±0.06	13.3	14.5
V11-168	0.52±0.10	20.1	14.9
V6 -140	0.95±0.05	152.6	15.0
V2 -110	0.98±0.10	2.4	15.3
V9 -44	0.41±0.06	31.4	16.5
V10-329	0.37±0.05	171.6	16.7
V6 -189	0.55±0.04	9.4	16.9
V8 -32	0.51±0.06	5.2	17.4
V6 -75	1.53±0.06	78.7	17.6
V2 -66	0.34±0.06	3.3	17.8
V2 -113	0.51±0.06	12.8	18.3
V6 -137	0.71±0.05	8.1	18.5
V12-136	0.63±0.07	32.9	19.1
V4 -479	0.90±0.05	70.3	19.4
V6 -47	0.42±0.09	7.3	19.8
V4 -209	0.25±0.07	10.7	20.5
V4 -436	0.24±0.05	30.5	21.3
V4 -207	0.49±0.05	18.8	21.7
V6 -211	0.53±0.08	3.7	22.3
V9 -23	1.50±0.05	177.8	22.8
V11-210	0.75±0.04	100.6	24.6
V2 -209	0.20±0.05	68.2	24.9
V6 -488	0.88±0.04	17.8	27.0
V6 -533	0.28±0.03	68.5	29.2
V11-32	0.54±0.06	69.8	29.3
V11-35	0.51±0.02	25.6	30.0
V4 -140	0.19±0.02	16.2	30.8
V6 -37	0.27±0.03	38.0	31.8
V9 -34	0.47±0.03	10.7	32.6
V4 -137	0.39±0.03	39.8	34.0
V6 -50	0.93±0.04	24.1	34.4
V6 -235	0.25±0.03	27.3	37.0
V10-12	0.17±0.04	3.7	39.1
V6 -410	0.35±0.04	11.1	39.9
V4 -93	0.25±0.03	32.3	41.0
V2 -97	0.25±0.02	169.6	41.5
V11-97	0.14±0.03	83.5	41.5
V6 -62	1.18±0.04	3.8	42.6
V10-333	0.16±0.03	25.1	43.2
V11-156	0.51±0.04	23.4	46.7
V10-30	0.31±0.03	10.5	47.1
V6 -111	0.30±0.02	60.4	49.8
V11-205	0.24±0.02	81.1	54.8
V6 -433	0.35±0.02	40.0	55.8
V11-47	0.43±0.04	112.0	57.0
V10-48	0.38±0.02	32.9	61.0
V6 -391	0.73±0.02	4.3	72.2

[a] The opening angles of all observed events were measurable. The error in the opening-angle measurement is of the order of 1 deg. Therefore, small angles are measured with rather large percentage errors, and their distribution is unreliable. The π^0 path lengths could be measured accurately (the path length exceeding the error of its measurement by a factor >3.5) for $s\alpha > 4$ μ-deg. However, the pairs could not be found reliably unless the angle α was less than approximately 50 to 70 deg.

[b] These distances, labeled events of type (b), are not considered measurements because the errors are comparable to the gap lengths. They were used only indirectly in the π^0 mean life evaluation. Correction for the bias has been made. See text for more discussion.

[c] Labeled type (a) in the text, these π^0 path lengths were not measurable because there was no way of determining the star center.

[d] The path of the neutral pion could not be defined.

by Dalitz.[8,16] (See the Appendix for the derivation of the formulas and the procedure followed in obtaining the expected opening-angle distribution in the laboratory system.) The points in the figure are the actual values obtained in the calculations. The smooth curve is passed through these points. The theoretical points are normalized to the number of events with opening angles between 10 and 50 deg; in this interval it is judged that few events were missed.

[16] Yu. A. Badagov, S. Wiktor, V. P. Dzhelepov, P. E. Ernolov, and V. I. Moskalev, Zh. Eksperim. i Teor. Fiz. 38, 1047 (1960) [English transl.: Soviet Phys.—JETP 11, 755 (1960)].

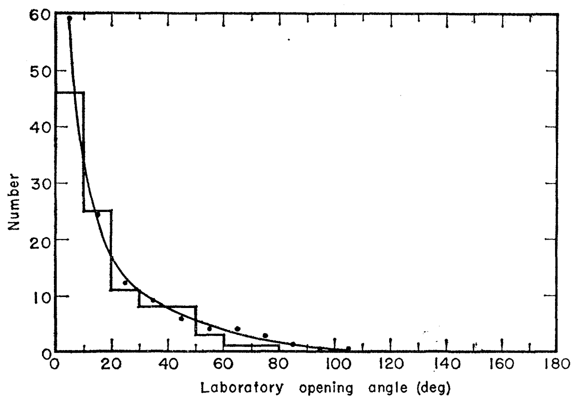

FIG. 2. Laboratory-system opening-angle distribution of all observed Dalitz pairs from π^0 decay. The smooth curve is the theoretical distribution. More description is given in the text.

Figure 3 shows the distribution of the angle between the plane containing (e^+,e^-) pair and the emulsion plane. The angle is measured from the emulsion (horizontal) plane. The distribution is expected to be isotropic from 0 to 90 deg. From the figure we see that any possible deviation is not statistically significant.

In principle, the π^0 momentum is measurable, and, in fact, it was measured for a number of individual events. We chose a statistical approach, however, to evaluate the mean life of the π^0. In this way we could use all of our π^0-gap data. We have assumed the π^0 momentum spectrum to be the same as that of the charged pions (which we found by multiple-scattering measurements). Justification for this assumption is given below.

Figure 4 shows the momentum spectrum of secondary charged pions within a cone of half-angle 60 deg relative to the primary negative pion. The transverse momentum distribution for the secondary charged pions emitted at all angles is shown in Fig. 5. The smooth curve is normalized to the total number of secondaries measured. It is best fit to the measured distribution with a function of the form $f(p_T)=Kp_T\exp[-p_T/p_0]$. The mean transverse momentum is 274 ± 10 MeV/c.

The total number of events we have observed (measured and unmeasured), plus the number missed of type (c), can be compared with that expected. The mean multiplicity of the secondary charged pions is 4.6. We assume that approximately half as many secondary neutral pions were produced as charged ones.[17–21] Therefore, with the experimental branching ratio of 0.01166 ± 0.00047 for the Dalitz mode,[16,20–24] we would expect to find a total of 97 ± 15 events at all angles, and 73 ± 9 events within the 60-deg cone.

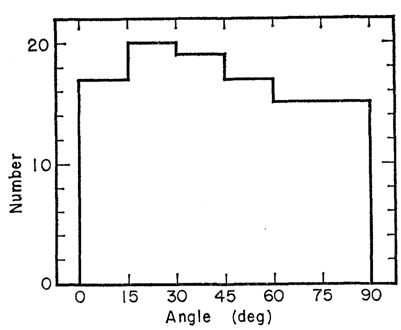

FIG. 3. Distribution of the angle between the plane containing the Dalitz pair and the emulsion (horizontal) plane.

The comparison is as follows:

	All angles	Within 60-deg cone
Measured	75	56
Additional observed	28	19
Type (c) correction	6	4
Total	109	79
Expected	97 ± 15	73 ± 9

It should be noted that the events of type (c) were estimated by considering the area shown in Fig. 2 between the toe of the calculated curve and the observed histogram.

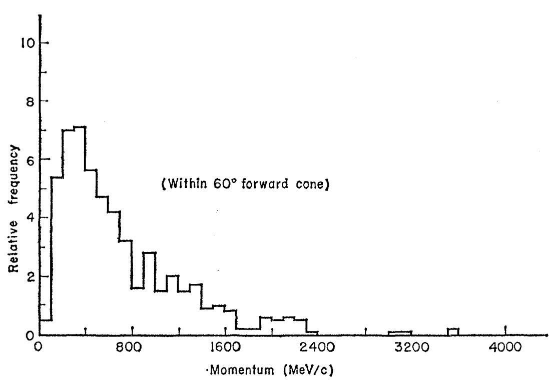

FIG. 4. Momentum distribution of secondary charged pions within 60-deg forward cone.

The expected numbers agree rather satisfactorily with those observed. This helps to justify our assumption that the momentum spectrum of the secondary charged pions is similar to that of the neutral pions. Further discussion on this point is given below.

[17] A. G. Carlson, J. E. Hooper, and D. T. King, Phil. Mag. **41**, 701 (1950).
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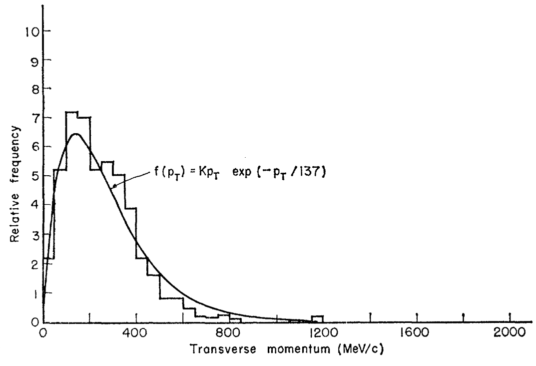

FIG. 5. Transverse momentum distribution of secondary charged pions at all angles.

B. Evaluation of the Neutral Pion Mean Life

The probability $P(s)$ that the π^0 path exceeds a distance s (cm) before decay is given by

$$P(s)=\int_0^\infty [\exp(-s/c\eta\tau)]f(\eta)d\eta, \qquad (1)$$

where τ is the proper mean life of the π^0 in seconds, η is the ratio of the pion momentum in MeV/c to its rest energy in MeV, and c is the velocity of light in cm/sec. The normalized momentum spectrum $f(\eta)d\eta$ was found as described above.

Of course the measured gaps contain the uncertainties in the points of origin of the tracks. To calculate analytically the errors in our determination of the decay point of the π^0 is extremely difficult. Therefore, we have resorted to "Monte Carlo" calculations for this purpose.

For the realistic generation of tracks one must put into the program the grain noise and the multiple scattering, both of which have Gaussian distributions.[25] The grain spacing also is a random variable, the distribution being closely exponential in these near-minimum tracks. The mean values and variances of the necessary parameters were obtained by measurements on the actual tracks of the electron pairs. These parameters were introduced into IBM-7094 subroutines in order to generate coordinates of grains in tracks by the Monte Carlo method.

We let each track be defined by six grains. The electrons were started from the origin and grains were successively generated by the Monte Carlo process mentioned above. A linear least-squares fit was made to each six-grain track.

We then chose pairs of tracks and varied the opening angles between them. Their common point of origin was taken at (0,0). However, the multiple scattering and noise variables of the grains caused the lines fitted to the tracks to intersect at some other point (x_i,y_i). This point was calculated for the different opening angles.

[25] W. H. Barkas, *Nuclear Research Emulsions* (Academic Press Inc., New York, 1963), Vol. I, Sec. 8.4.

For the calculation of the mean life of the π^0 we employed the experimentally observed distributions of:

(a) the charged pion momentum (assumed the same for the neutrals),
(b) the laboratory opening angles of the Dalitz pairs,
(c) the emission angles of the charged pions with respect to the direction of the incoming π^- (assumed the same for π^0).

We selected several values of the mean life (τ_{π^0}), and by Monte Carlo calculations we found the direction of the π^0 (with respect to the incoming π^-) and its point of decay with respect to the parent star. We included the error brought in by assuming the intersection of the members of the pair to be the π^0 decay point.

From these calculations we determined the integral gap-length distribution and the angular distribution for each value of τ_{π^0}. These distributions were then compared to that obtained from the measured neutral pions. At the limit of vanishing π^0 mean life, the latter distribution is determined solely by the errors in determining the π^0 production and decay points.

As described in Sec. III A, we limited the mean-life determination to those measured π^0 events which were emitted in a forward cone of 60 deg half-angle. The integral path-length distribution of the 56 measured π^0 events emitted in this region gave the best χ^2 fit at

$$\tau_{\pi^0}=(1.7\pm0.3)\times10^{-16} \text{ sec.}$$

The confidence interval corresponds approximately to ±1 standard deviation. It encompasses two-thirds of the area under the curve of the distribution function, one-third above the mode and one-third below.

Figure 6 shows the χ^2 distribution for various assumed values of τ_{π^0} used in the calculation of integral path length distribution. Figure 7 shows the experimental points and those calculated by the Monte Carlo method with the most probable τ_{π^0} value.

The integral path length distribution of the 75 measured events, emitted in all directions, gave the best χ^2 fit at

$$\tau_{\pi^0}=(1.8\pm0.5)\times10^{-16} \text{ sec.}$$

The observed angular distribution of the π^0 mesons should be the same as that of the charged pions except for the folded-in error in determining the point of decay of the π^0.

On excluding the short gaps ($<0.1\,\mu$) which were usually not detected in this experiment, the machine-generated angular distributions were compared with that observed. We believe this gives us an almost independent estimate of the π^0 mean life.

The most probable mean life resulting from this comparison (75 measured events) is

$$\tau_{\pi^0}=(1.6\pm0.4)\times10^{-16} \text{ sec.}$$

C. Treatment of Systematic Errors

We now list some important sources of systematic errors, their effects on the mean life evaluation, and corrections for them.

1. *Momentum Spectrum of the Secondary Charged Pions*

The movement of the individual tracks were assumed with error $\leqslant 20\%$. By having two operators measure different segments of the same track, we were able to establish the limit. For another error estimation, we measured some 100 primary π^- tracks.

We assume that the true distribution of the sagitta (mean second difference D) is distorted by a Gaussian-like distribution with variance σ^2. We can then calculate from the relation

$$2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(D_1 - D_2)^2,$$

where D_1 and D_2 are the values of the sagitta obtained by two operators for different segments of the same track. We get $\sigma = 0.006\ \mu$.

The distribution of the measured values of D for the secondary charged pions can be approximated by the sum of two Gaussian-like distributions; for each, the standard deviation is about $0.3\ \mu$.

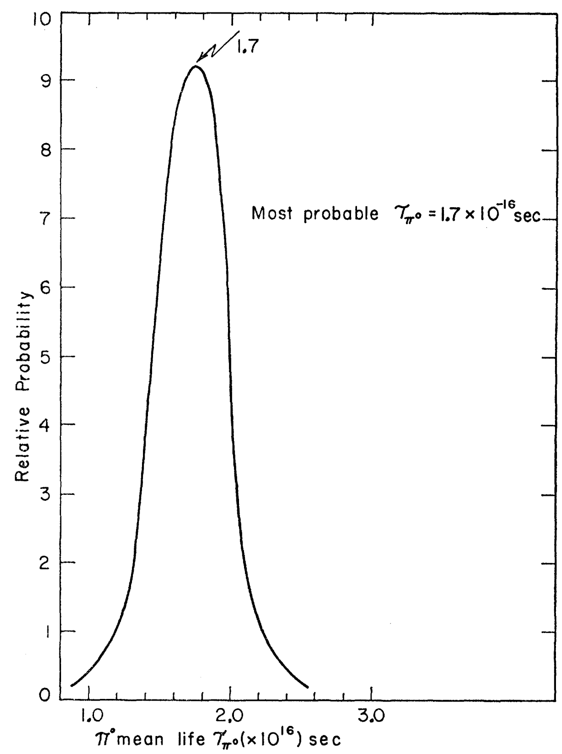

FIG. 6. χ^2 distribution for various assumed values of τ_{π^0} used in the calculation of integral path-length distribution of π^0 by the Monte Carlo method.

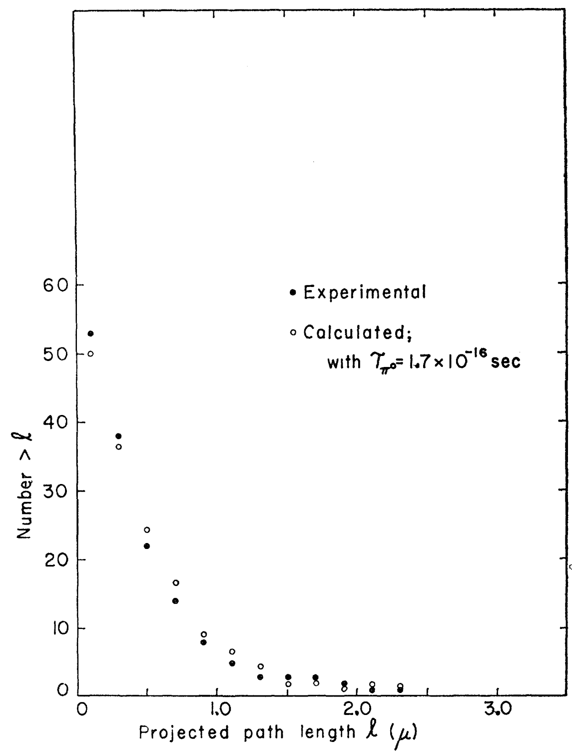

FIG. 7. Integral distribution of projected π^0 path lengths. Solid points are experimental, and open points are calculated by the Monte Carlo method (mentioned in the text) with most probable τ_{π^0} value.

It can be shown that the product of two Gaussian functions, with variances σ^2 and σ'^2, is another Gaussian function with variance Σ^2 given by $\Sigma^2 = \sigma^2 + \sigma'^2$.

To see the effect of this fluctuation (in the measured D distribution) on the mean-life evaluation, we use Eq. (1) for illustration.

For constant cell length t we can write $\eta \approx K_1/D$. K_1 is a slowly varying function of the velocity β and the cell length t. For the measured relativistic pions with β ranging from 0.95 to 1.0, we can assume that K_1 is approximately constant for fixed t.

Consider one of the two Gaussian distributions (which approximate the true D distribution) to be

$$\exp[-(D - \langle D\rangle)^2/2\sigma'^2].$$

Therefore, for this part of the D distribution, Eq. (1) becomes

$$P(s) \approx \int \left[\exp\left(\frac{-sD}{c\tau K_1}\right)\right]\left[\exp\left(\frac{-(D-\langle D\rangle)^2}{2\sigma'^2}\right)\right] D^{-2} dD.$$

We assume that the true D distribution is distorted by a Gaussian-like distribution with variance σ^2.

Therefore, because of fluctuations in the measurements of D, σ' in the above equation has to be replaced by Σ, where

$$\Sigma \approx \sigma'(1 + 1/5000).$$

The fluctuations in the multiple-scattering measurements introduce insignificant effects on the mean-life evaluation.

2. *Assumption that the Momentum Spectrum of the Secondary Charged Pions is the same as that of the Neutral Pion*

Figure 1 shows that the projected angle of the secondary charged pions is very similar to that of the neutral pions. This gives us the first piece of evidence that there is great similarity in the production of $\pi^\pm$ and π^0. In experiments on multiple pion production by negative pions incident on emulsion nuclei, it has been found that the ratio of the secondary charged to neutral pions is very close to 2.0.[21] This ratio was also obtained in several cosmic-ray experiments.[17–20] As mentioned before, the number of π^0 events expected agrees satisfactorily with the number we observed.

We, therefore, believe that no large error is introduced by our assumption regarding the momentum spectrum.

3. *General Measurement Errors*

These include errors due to individual operators, and variation in the room temperature and humidity. In order to eliminate these effects the emulsion plate was put into temperature and humidity equilibrium with the surroundings before any measurement began. Each event was measured by two operators, often on different days. It was required that the two results agree within quoted errors. When a π^0 event was found, each value of the π^0 path was weighted by its error and the two values combined to give the resulting π^0 path and its combined error.

The Koristka MS2 microscope light was turned on for at least an hour prior to making any measurements. The light itself was located external to the thermal insulation surrounding the microscope. A breath shield was also used. These precautions against thermal drift reduced it to less than 0.003 μ per min. Since only ***relative*** distances are involved, and the relative coordinates of all the grains in an event were measured during a period of 2–5 min, the thermal drift of the microscope was not large enough to affect the measurements.

4. *Measurement Precision and Scanning Efficiency*

Out of 3600 interaction stars, the first two measurements were in disagreement or ambiguous in about 80 cases. These events required measurement for the third and fourth time (by different operators). In all cases, we were able to come to some conclusion after the second set of measurements.

A third measurement was also made on each of a sample of five randomly picked π^0 events. Each of these measurements was found to agree well with the first two.

To check on the efficiency of the scanners in observing secondary minimum-ionizing tracks, a fair sample of about 200 stars was rescanned. Some 15 additional secondary "minimums" were found, with not more than one additional for each star. None of these additional "minimums" affect the near-100% efficiency in the observation of π^0 events. Pairs are usually more conspicuous than lone minimum tracks.

5. *Contamination of e^+, e^- Pairs from γ Conversion and Heavy-Particle Decay*

The average γ-conversion length in emulsion is about 4 cm. The mean multiplicity for secondary charged pions is 4.6. We assume that approximately half as many secondary neutral pions were produced as charged.[17–21] The largest π^0 path length measured was 2.31 μ. Therefore, the number of γ-conversion pairs within 2.31 μ of the interaction points would be $(4.6\times 3600)\times 2.31/40\,000\approx 1$. This is $<1\%$ of the total number of Dalitz decay π^0 events observed.

There are decay modes of K^0 mesons and Λ hyperons that have only electron pairs and neutral particles as products. However, their contamination of electron pairs near the primary interaction points is negligible.

6. *Unmeasured Events of Type (a)*

The total number of unmeasured stars of type (a) is 10 (see Sec. IIIA). These events were not measurable because we had no way to determine the point of production of the π^0. We believe that the π^0 path-length distribution for these events must be very similar to a typical sample of all the events observed. We estimated the error introduced by these unmeasured events to be $\lessapprox 20\%$ in our mean-life evaluation.

Combining all the errors discussed above (1 through 6), we estimated a maximum over-all systematic error of approximately 30% in our mean-life evaluation.

Therefore, the value of the π^0 mean life, as determined in this experiment, is

$$\tau_{\pi^0}=(1.7\pm 0.5)\times 10^{-16}\text{ sec.}$$

IV. DISCUSSION

The mean life of the π^0 as found in this experiment is in good agreement with the published values.[3–7,15,26,27] They are:

	τ_{π^0} (sec)
Blackie *et al.*[3]	$(3.2\pm 1.0)\times 10^{-16}$
Glasser *et al.*[4]	$(1.9\pm 0.5)\times 10^{-16}$
Tietge *et al.*[5]	$(2.3_{-1.0}^{+1.1})\times 10^{-16}$
Koller *et al.*[6]	$(2.8\pm 0.9)\times 10^{-16}$
Evans *et al.*[7]	$({}_{<2.2}^{>0.5})\times 10^{-16}$
Davidson[15]	$(1.5_{-1.0}^{+10.0})\times 10^{-17}$
Ruderman[26]	$(7.6_{-4.5}^{+6.0})\times 10^{-17}$
Von Dardel *et al.*[27]	$(1.05\pm 0.18)\times 10^{-16}$

[26] H. Ruderman, thesis abstract, California Institute of Technology, 1962 (unpublished).

[27] G. Von Dardel, D. Dekkers, R. Mermod, J. D. Van Putten, M. Vivargent, G. Weber, and K. Winter, Phys. Letters 4, 51 (1963).

The first five experiments were carried out by using neutral pions from the $K_{\pi 2}$ decay ($K^+ \to \pi^+ + \pi^0$), which produce path lengths of the π^0 just at the limit of measurability. The sixth and seventh experiments used the Primakoff effect of producing π^0 by one incident γ and another γ from the Coulomb field of a nucleus. The experiment of Von Dardel *et al.*[27] was done by counter detection of positrons from the decay of neutral pions. The magnitude of the π^0 mean life we have found is in most serious disagreement with their measurement. They employed quite a different method from ours, and it is probable that the same quantity is not being measured in the two experiments, or equivalently that one or the other of the experiments contains unrecognized systematic effects.

There is some evidence, recently published, indicating that the value of τ_{π^0} lies near our measured value. Wong has calculated the decay of the neutral pion by extending the process $\gamma+\pi \to \pi+\pi$ to $\gamma+\pi \to \pi+\pi \to \gamma$.[28] He treated the dominant intermediate state as that of 2π, rather than N-$\bar{N}$ pairs only—as considered by Goldberger and Treiman.[29] With the parameter $\Lambda = 1.8$ obtained by Ball (in the paper on the application of the Mandelstam representation to photoproduction of pions from nucleons),[30] and using Wong's calculation, we get $\tau_{\pi^0} \approx 2.2 \times 10^{-16}$ sec.

On the basis of a semiclassical model of the Bohr-Sommerfeld type, Sternglass has investigated the relativistic electron-pair system in the limit of high velocities.[31] He showed that a lowest state exists. It possesses an energy approximately equal to the π^0 rest energy. The calculated π^0 rest energy is 134.4 MeV, compared to the measured value of 135.0 MeV.[32] Next, he showed that the lifetime of the system against annihilation into two photons is 2.06×10^{-16} sec, which is in good agreement with the value we have found.

Our experimental value for the π^0 mean life disagrees, however, with that calculated[33] on the assumption that the π^0 decay is dominated by the process[34] $\pi^0 \to \rho^0 + \omega^0$ followed by $\rho^0 \to \gamma$, $\omega^0 \to \gamma$. The calculated value π_{π^0} for this process is $\approx 6 \times 10^{-18}$ sec. According to this scheme, the branching ratio $(\rho \to \eta\pi)/(\rho \to 2\pi)$ is expected to be $\approx 1/4$, whereas the experimental value is $<1\%$[35] and is consistent with zero.

In the recent experimental data of King there seems to be some deficiency of Dalitz pairs in the vicinity of the star centers.[36] Within 10 μ of the star centers he observed only three Dalitz pairs, compared with the expected number of eight pairs. The probability that this disagreement is due to purely statistical fluctuation is about 4%. We believe, however, that the deficiency in the number of observed pairs arises from the scanning (or measurement) inefficiency. Without determining the points of intersection of all pairs of relativistic particles by a technique such as ours, a large fraction of the pairs will certainly be missed. A bias exists especially against finding pairs of large opening angle.

ACKNOWLEDGMENTS

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APPENDIX: CALCULATION OF THE LABORATORY-SYSTEM OPENING-ANGLE DISTRIBUTION OF DALITZ PAIRS

(For all measured values of η)

Figure 8 shows the orientation of the Dalitz decay (e^+,e^-) in the rest frame of the π^0 (called c.m. below)—quantities relative to this reference frame have asterisks. Quantities without asterisks refer to the laboratory frame.

We assume that the e^+ in the c.m. system and the π^0 motion in the laboratory system (along x^* axis) lie in the x^*y^* plane. Therefore we see that u^* can have any value between 0 and 2π with equal probability, and $\cos\theta_+^*$ can have any value between -1 and $+1$ with equal probability.

The relation between $\theta_\pm$ and $\theta_\pm^*$ is given by

$$\tan\theta = \frac{\sin\theta^*}{\gamma_0[\cos\theta^* + (\beta_0/\beta^*)]}. \tag{A1}$$

Since the electron rest energy of approximately 1/2 MeV is normally much smaller than its total energy in the c.m. system, we can take $\beta^* \approx 1$. We have $\beta_0(1-\beta_0)^{-1/2} = \eta = p/m$, where p and m are momentum (MeV/c) in the laboratory system and rest energy (MeV) of the

[28] How-sen Wong, Ph.D. thesis, Lawrence Radiation Laboratory Report UCRL-9251, 1960 (unpublished).
[29] M. L. Goldberger and S. B. Treiman, Nuovo Cimento **9**, 451 (1958).
[30] James S. Ball, Phys. Rev. **124**, 2014 (1961).
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[32] W. H. Barkas and A. H. Rosenfeld, Lawrence Radiation Laboratory Report UCRL-8030 (Rev), 1964 (unpublished).
[33] M. Gell-Mann, D. Sharp, and W. G. Wagner (to be published).
[34] For the ρ meson: A. Erwin, R. March, W. Walker, and E. West, Phys. Rev. Letters **6**, 628 (1961); for the ω meson: B. Maglić, L. Alvarez, A. Rosenfeld, and M. Stevenson, *ibid.* **7**, 178 (1961); for the η^0 meson: A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein *et al.*, M. Block, A. Engler, R. Gessaroli, and C. Meltzer, *ibid.* **7**, 421 (1961).
[35] A. H. Rosenfeld (private communication). For the Research Progress Meeting, 22 February 1962, Dr. Rosenfeld gathered all experimental data available (as of 22 February 1962) and found this experimental branching ratio.
[36] D. T. King, University of Tennessee Report, 1962 (unpublished).

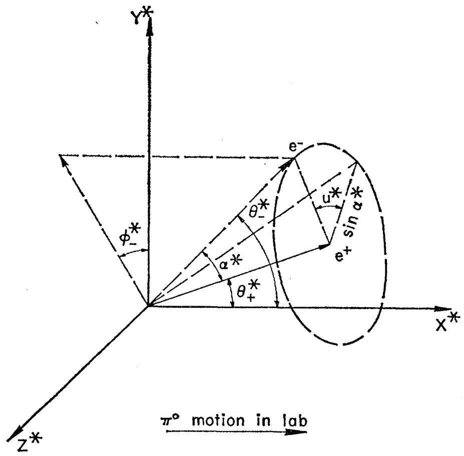

FIG. 8. Orientations of the Dalitz (e^+,e^-) in the rest frame of the π^0.

π^0. Then the opening angle α (in the laboratory system) of the pair is found from

$$\begin{aligned}\cos\alpha=&\pm[(1+f)(1+g)]^{-1/2}\\&\pm[1-(1+f)^{-1}]^{1/2}[1-(1+g)^{-1}]^{1/2}\\&\times[1-(1-a^2)^{-1}(\sin^2\alpha^*\sin^{-2}u^*)]^{1/2},\quad\text{(A2)}\end{aligned}$$

where

$$a=\cos\theta_-^*,$$

$$f=\sin^2\theta_+^*[(1+\eta^2)^{1/2}\cos\theta_+^*+\eta]^{-2},$$

and

$$g=(1-a^2)[(1+\eta^2)^{1/2}a+\eta]^{-2}.$$

To find the theoretical distribution of α, we proceed as follows:

As mentioned above, the distributions in $\cos\theta_+^*$ and u^* are isotropic from -1 to $+1$ and from 0 the 2π, respectively. The weighting factor for each α corresponding to each set of α^*, u^*, η, and $\cos\theta_+^*$ is simply

$$f(\eta)\exp(-a^*/\alpha^*)\alpha^{*-1},$$

where $f(\eta)$ is the measured momentum spectrum described in the text. The $\alpha^{*-1}\exp(-a^*/\alpha^*)$ is an empirical fit to the theoretical opening-angle distribution of the e^+, e^- pair, as calculated by Dalitz.[8,16] In the text this distribution is denoted by $q(0,\alpha)$. According to Joseph (quoted in Ref. 23), for normalization we have taken the distribution to approach zero linearly from $\alpha^*\approx 100$ to 180 deg. With 18.1 deg for the median angle,[16] we obtained $a^*\approx 1.8$ deg.

In our calculation, we first picked eight equal intervals for each of the four variables α^*, u^*, η, and $\cos\theta_+^*$. To improve the calculation we next picked, for each, another set of intervals that lie halfway between the previous set.

Physically, the $\pm$signs mean that the pair can be in any of the octants. We know experimentally that the majority of the opening angles have $\cos\alpha\gtrless 0$.

In our calculation, we took all possible combinations of the $+$ and $-$ signs, and let the four variables take all possible values at equal intervals (as described above).

We have seen that the theoretical distribution of laboratory-system opening angles (for all values of η) is not much different from that in the rest frame of the π^0. This fact enables us to assume that the distribution of laboratory-system opening angles falls as $\alpha^{-(1+\epsilon)}$ for large angles, where ϵ is a function of η. Using the calculations above, we can determine (as a first approximation) ϵ and $\alpha_{1/2}$ as function of η. The $\alpha_{1/2}$ is the median laboratory-system opening angle for each η. Next, to take care of the distribution at small angles (for the limited region of η in our calculation), we assume the laboratory-system distribution of opening angles to be

$$q(\alpha,\eta)=[\exp(-a/\alpha)]\alpha^{-(1+\epsilon)}d\alpha,$$

where a is a function of η. This distribution reduces to the distribution

$$[\exp(-a^*/\alpha^*)]\alpha^{*-1}d\alpha^*,$$

for $\eta=\epsilon=0$.

With the previously obtained values of $\alpha_{1/2}$ and ϵ as functions of η, we can find the values of a (as a function of η also) from the relation

$$\int_{\alpha_{1/2}}^{\alpha_{\max}}[\exp(-a/\alpha)]\alpha^{-(1+\epsilon)}d\alpha=\int_0^{\alpha_{1/2}}[\exp(-a/\alpha)]\alpha^{-(1+\epsilon)}d\alpha.$$

We took $\alpha_{\max}\approx 100$ deg; beyond this angle we have observed none. Any deviations from the results for $\alpha_{\max}=180$ deg will be only second- or third-order corrections.

TABLE II. Computed values of ϵ and a for various values of η.

η	ϵ[a]	a (deg)[b]
0.0	0	1.7
3.0	0.006	1.0
6.0	0.012	0.6
9.0	0.018	0.4
12.0	0.024	0.22
15.0	0.030	0.13
18.0	0.036	0.08
21.0	0.042	0.05

[a] $\epsilon\approx 0.002\eta$.
[b] $a\approx 1.7\exp(-0.17\eta)$.

Iterated processes then give us the best values of ϵ and a as functions of η. Their computed values for various values of η are given in Table II.